(fatigue data from Tables XLII and XLIII for 10^6-10^7 cycles life), under the following conditions:

$$(\sigma_\theta)_{max} = 0, \quad (\sigma_\theta)_{min} \geq -\sigma_1 \tag{88}$$

Equation (84) and the definition

$$(\sigma_\theta)_{min} = (\sigma_\theta)_{max} - 2(\sigma_\theta)_r$$

require from (85) that

$$(\sigma_\theta)_{min} = -2/3\,\sigma_1 \quad . \tag{89}$$

To obtain conditions (87-89) a fluid-support pressure varying between q_1 and p_1 is to be found. Because the inner unit consists of only one ring in this case, calculations on the computer are not necessary as they are easily performed by hand. The analysis proceeds as follows:

$$(\sigma_\theta)_{max} = p_o \frac{K^2 + 1}{K^2 - 1} - 2p_1 \frac{K^2}{K^2 - 1} = 0 \quad ,$$

$$p_1 = \frac{p_o}{2} \frac{K^2 + 1}{K^2} = 325,000 \text{ psi} \quad , \tag{90}$$

$$(\sigma_\theta)_{min} = -2q_1 \frac{K^2}{K^2 - 1} = -2/3\,\sigma_1 \quad ,$$

$$q_1 = \frac{K^2 - 1}{K^2} \frac{\sigma_1}{3} = 55,500 \text{ psi} \quad . \tag{91}$$

Thus, it is found that the outer unit must withstand an internal pressure varying between 55,500 psi and 325,000 psi.

The computer code, MULTIR, is used for the outer-unit calculations. A 1/2-inch gap is allowed between the units for the fluid-support pressure, i.e., $r_o = 4.50 + 0.50 = 5.00$ in. for the outer unit. The assumed data are

wall ratio, $K = 4.0$,

number of rings, $N = 3$,

ring radii, $r_o = 5.0$ in., $r_1 = 7.95$ in., $r_2 = 12.61$ in., $r_3 = 20.0$ in.,

support pressures, $p_N = q_N = 0$,

minimum bore pressure, $q_o = 55,500$ psi,

fatigue coefficients, $A_n = 2.86$, $B_n = 1.14$.

Different calculations, 1A - 1D, are performed for rings made from materials with various strengths. Results are given in Table XLV. All four calculations give results

that satisfy the requirement of maximum bore pressure of p_o = 325,000 psi. The effect of varying the strength of the rings is indicated. Design 1B has the minimum required interference, Δ_1 = 0.0622 in., corresponding to $\frac{\Delta_1}{r_1} = \frac{0.0622}{7.95} = 0.00782$ in. in.

TABLE XLV. RESULTS OF COMPUTER CODE MULTIR FOR EXAMPLE DESIGN 1[a]

Design	Design Tensile Strength of Rings, σ_1, psi			Results		
	1	2	3	Maximum Bore Pressure for 10^6 Cycles Life	Required Interference[b], in. Δ_1	Δ_2
1A	325,000	325,000	325,000	338,337	0.0670	0.0739
1B	350,000	325,000	300,000	332,699	0.0622	0.0630
1C	375,000	350,000	300,000	345,837	0.0658	0.0578
1D	400,000	350,000	300,000	351,251	0.0625	0.0578

(a) Based entirely on the tensile-fatigue criterion.

(b) Interferences required on the radius. Δ_1 required between rings 1 and 2, and Δ_2 required between 2 and 3.

Example Design 2

In this design the more conservative shear-fatigue-strength criterion is used for the outer (second) ring of the inner unit and for all three rings of the outer unit. The given data are:

Inner Unit

wall ratio, K = 3,

number of rings, N = 2,

radii, r_o = 3.00, r_1 = 5.1960, r_2 = 9.00,

tensile strength of ring 1, σ = 300,000 psi,

yield strength of ring 2, σ_y = 212,500 psi (σ_y = 0.85 σ_u, σ_u = 250,000 psi),

fatigue coefficients,

A_1 = 2.86 and B_1 = 0. for ring 1,

A_2 = 2.55 and B_2 = 2.0 for ring 2,

minimum bore pressure, q_o = 0,

support pressures, p_2 = 160,000 psi, q_2 = 0.

Outer Unit

wall ratio, K = 4,

radii, r_o = 9.500 in., r_1 = 15.07 in., r_2 = 23.90 in., r_3 = 38.00 in.,

number of rings, N = 3,

yield strength of rings, σ_y = 255,000 psi (σ_y = 0.85 σ_u, σ_u = 300,000 psi),

fatigue coefficients of rings, A_n = 2.55, B_n = 2.00,